NOTATION

a_m, thermal diffusivity; A, silicon absorptivity; k, α_m, and λ, absorption, heat exchange, and thermal conductivity coefficients; δ, thickness of the silicon film; q_f, heat flux density; t*, duration of the light pulse; W, pulse energy; $\Theta = T(x, t)/T_0$, dimensionless temperature; $\xi = x/\delta$, dimensionless coordinate; h and τ, space and time steps; $Bi = \alpha_m \delta/\lambda$, Biot number; $Ki = Aq_d\delta/\lambda\ T_0$, Kirpichev number; $Bu = k\delta$, Bouger number; and $Fo = a_m t/\delta^2$, Fourier number.

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NUMERICAL METHOD FOR SOLVING THE COUPLED PROBLEM OF RADIATIVE–CONVECTIVE AND CONDUCTIVE HEAT TRANSFER

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The solution of the problem of complex heat transfer is reduced to a systematic solution of a system of nonlinear equations and the heat-conduction equations. A rapid iterative method is proposed for solving the system of equations.

The problem of heating and cooling of a system of bodies with a complex shape under conditions of radiation–convection heat transfer has not been adequately studied. The case when the bodies are heated in a regular regime was examined in [1]. For nonstationary processes, this assumption is not satisfied.

We shall examine a radiating volume V, surrounded by a system of opaque bodies. The surface F of the volume V consists of the surfaces of the bodies and of "liquid" boundaries, through which the heat carrier enters and leaves the volume. We shall view the latter as fictitious surfaces, allowing gas to pass freely through them. These surfaces are assigned a certain temperature (or flux density of the resulting radiation), as well as an effective emissivity. This artificial technique is used quite frequently [2] to close the emitting system in examining radiative transfer and permits the gas flow to leave the system at the same time.

We shall divide the volume V and the bounding surface F into N zones. For each zone n, we shall write the law of conservation of energy in the form

$$Q_{\text{rad.},n} = -Q_n,\ n = 1,\ 2,\ \ldots,\ N. \tag{1}$$

The radiant energy transport is approximated using the resolvent method by a system of algebraic equations

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$$Q_{\text{rad},n} = \sum_{m=1}^{N} A^{\Sigma}_{mn} T^4_m, \quad n = 1, 2, \ldots, N, \tag{2}$$

where A^{Σ}_{mn} are the selective radiative transfer coefficients introduced in [3]; T_m is the temperature of zone m. Without considering the details of the methods used for determining the coefficients A^{Σ}_{mn}, we note that they depend primarily on the geometry of the radiating system and to a lesser extent on the temperature field and the directional distribution of the radiation intensities. This permits limiting the analysis to periodic refinement of the coefficients A^{Σ}_{mn} without iteration, in solving the nonstationary problem.

In accordance with the simplified zonal method, we shall assume that within each volume zone, the gas is adequately mixed and is essentially isothermal. The total flux of enthalpy contained in the gas flowing into the volume zone n equals

$$Q^{+}_n = \sum_{q'} W_{q' \to n} (\bar{C}_p T)_{q'}, \tag{3}$$

while the flux of enthalpy contained in the gas flowing out of the volume zone n is

$$Q^{-}_n = W_n (\bar{C}_p T)_n, \tag{4}$$

where $\bar{C}_p$ is the average heat capacity of the gas in the temperature range 0-T°K.

The flow of heat into the volume zone n due to heat transfer by convection Q^{α}_n and turbulent heat conduction Q^{tr}_n is proportional to the difference of the temperatures of zone n and neighboring zones:

$$Q^{\alpha}_n = \sum_{m'} \alpha_{m'} F_{m'} (T_{m'} - T_n), \tag{5}$$

$$Q^{\text{tr}}_n = \sum_{q'} K_{nq'} F_{nq'} (T_{q'} - T_n). \tag{6}$$

It is evident from (3)-(6) that the resulting flow of heat into the volume zone n due to convection can be written in the form

$$Q^{+}_n - Q^{-}_n + Q^{\alpha}_n + Q^{\text{tr}}_n = \sum_{m=1}^{N} g_{mn} T_m, \tag{7}$$

where g_{mn} are the coefficients of convective heat transfer, which differ from zero only for neighboring zones m and n and depend in general on the temperatures of the zones. From here, in the absence of molecular heat conduction in the gas, we obtain for each volume zone n

$$Q_n = -(\rho C_p)_n V_n \frac{\partial T_n}{\partial \tau} + \sum_{m=1}^{N} g_{mn} T_m + Q_{\text{in},n}, \tag{8}$$

where ρC_p is the volume heat capacity of the medium; V_n is the volume of zone n; and $Q_{\text{in},\,n}$ is the internal liberation of heat in the zone (for example, due to combustion).

We shall now examine the surface zone m. In addition to radiation, heat enters it due to heat transfer and heat conduction and, in addition, at the solid body—gas boundary the following coupling conditions must be satisfied:

$$T_m(\tau) = \Theta_k(x, y, z, \tau)|_{F_m},$$
$$Q_m(\tau) = \alpha_m F_m (T_{q'}(\tau) - T_m(\tau)) - \int_{F_m} \lambda_k \frac{\partial \Theta_k}{\partial n} dF_m, \tag{9}$$

where $T_{q'}$ is the temperature of the volume zone q', adjacent to the surface zone m; $\Theta_k(x, y, z, \tau)$ is the temperature field in the k-th body, on whose surface zone m is situated. The functions $\Theta_k(x, y, z, \tau)$ are determined from a solution of the heat-conduction equations

$$\rho C_k \frac{\partial \Theta_k}{\partial \tau} - \nabla (\lambda_k \nabla \Theta_k) = 0 \tag{10}$$

with the coupling conditions (9) on the surfaces, referred to the volume, and boundary conditions of the first, second, third, and fourth kind on the remaining surfaces of the k-th body.

Let us proceed to construct the finite-difference analog of the system of equations (1), (2), and (8)-(10). We introduce a time step $\Delta\tau$, such that $\tau_i = \Delta\tau i$. We indicate by the integer indices i the values of quantities at times τ_i and by the fractional indices i + 1/2 the values of functions determined on the segment $[\tau_i, \tau_{i+1}]$.

We shall approximate the heat flux of each zone n, Q_n, and Q_{rad} in the temperature interval $[\tau_i, \tau_{i+1}]$ by a step functions in terms of the values of the temperatures of zones on the boundaries of the interval. We shall call the zone n, for which such an approximation is satisfied explicitly in terms of the known values of its temperature at the time τ_i, the C zone. In the opposite case, we shall call the n zone the P zone, if in approximating the heat fluxes in implicit form, the unknown value of its temperature at time τ_{i+1} is used. The explicit approximation leads to a simpler computational scheme, but in this case the time step $\Delta\tau$ must be small enough to neglect the changes in temperature in the C zones in the time interval examined. For this reason, it is useful to choose for the C zones heat-absorbing surfaces, whose temperature depends comparatively little on time. On the contrary, from considerations of stability, we must always view the gas zones as P zones. We note that the fictitious surfaces introduced above, through which gas enters the volume and leaves it, can be viewed either as P zones with a predetermined quantity $Q_n(\tau)$ or as C zones with a fixed temperature $T_n(\tau)$, depending on the circumstances.

For definiteness, in what follows, we shall assume that all surface zones are C zones and have the numbers n = 1, 2, ..., M, while the gas zones have the numbers M + 1, ..., N. In accordance with this, we obtain for the interval $[\tau_i, \tau_{i+1}]$ instead of relations (1), (2), and (8):

$$\sum_{m=1}^{N} A_{mn}^{\Sigma} T_m^{4(l)} = -Q_n^{i+1/2}, \quad n = 1, 2, \ldots, N, \tag{11}$$

$$Q_n^{i+1/2} = -(\rho C_p)_n V_n \frac{T_n^{i+1} - T_n^i}{\Delta\tau} + \sum_{m=1}^{N} g_{mn} T_m^l + Q_{\text{in},n}^{i+1}, \quad n = M+1, \ldots, N. \tag{12}$$

Here $T_m^l = T_m^i$ with $m \leqslant M$ and $T_m^l = T_m^{i+1}$ with $m > M$.

Let us write the system of heat-conduction equations (10) in the interval examined:

$$\rho C_k \frac{\partial \Theta_k^{i+1/2}}{\partial\tau} - \nabla(\lambda_k \nabla \Theta_k^{i+1/2}) = 0, \tag{13}$$

where k is the number of the body. We shall determine the boundary conditions for (13) on surfaces, referred to the volume V, by approximating the coupling conditions (9):

$$\lambda_k \left.\frac{\partial \Theta_k^{i+1/2}}{\partial n}\right|_{F_m} = \alpha_m (T_{q'}^{i+1} - T_m^i) - Q_m^{i+1/2}/F_m, \tag{14}$$

$$T_m^{i+1} = \Theta_k^{i+1/2}(\tau_{i+1})|_{F_m}, \quad m = 1, 2, \ldots, M. \tag{15}$$

Here, it is assumed that the dimensions of the zone F_m are so small that the quantity $\partial\theta_k/\partial n$ remains practically constant within zone m. On the remaining surfaces of the bodies, the boundary conditions are determined in the usual manner. We shall give the initial conditions for the equations of heat conduction (13) in the form

$$\Theta_k^{i+1/2}(x, y, z, \tau_i) = \Theta_k^{i-1/2}(x, y, z, \tau_i).$$

Let us formulate the scheme for solving the system of equations (11)-(13) with the boundary conditions (14), (15) and the initial conditions:

$$T_m^{(0)} = T_{0,m}, \; m = 1, 2, \ldots, N; \; \Theta_k^{(1/2)}(x, y, z, 0) = \Theta_{0,k}(x, y, z),$$

where k enumerates the bodies.

1. Knowing the temperature field at time $\tau = 0$, we determine the coefficient of radiative transfer A_{mn}^{Σ}.
2. We solve the system of N−M nonlinear equations (11) and (12) for the temperature of the P zones

$$\sum_{m=1}^{M} A^{\Sigma}_{mn} T_m^{4(0)} + \sum_{m=M+1}^{N-M} A^{\Sigma}_{mn} T_m^{4(1)} = -Q_n^{(1/2)}, \tag{16}$$

where n = M + 1, ..., N, while the quantity $Q_n^{(1/2)}$ is defined in (12). As a result, we find $T_n^{(1)}$ for n = M + 1, ..., N.

3. We determine the quantity $Q_n^{(1/2)}$, n = 1, 2, ..., M, for the C zone from (11).

4. Since after this the boundary conditions (14) in the interval [0, $\Delta\tau$] are determined, we solve the system of equations of heat conduction (13).

5. We determine the temperatures of the C zones $T_n^{(1)}$, n = 1, 2, ..., M, at the times $\Delta\tau$ from the conditions (15).

6. If necessary we refine the coefficients of radiative transfer A^{Σ}_{mn} and repeat operations 2-5 at subsequent times.

Thus, at each time step $\Delta\tau$, it is necessary to solve a system of nonlinear equations (16). The gradient methods usually used for this purpose require $\sim(M-N)^3$ operations for each iteration. Since this system must be solved repeatedly, already for a comparatively small number of P zones ($\sim$30) the amount of computation turns out to be too large for modern computers.

Let us write the system of nonlinear equations (16) for the step i in vector form

$$\|A\| \vec{T}^{4(i+1)} = \vec{P}^{(i)} - \vec{Q}^{(i+1/2)}. \tag{17}$$

Here $\|A\| = [A^{\Sigma}_{mn}]$ is the matrix of radiative transfer coefficients for the C zones with dimensions $(M-N) \times (M-N)$; $\vec{T}^{4(i+1)} = [T_n^{4(i+1)}]$, $\vec{Q} = [Q_n^{i+1/2}]$, n = M + 1, ..., N, while the components of the vector $\vec{P}^{(i)}$ equal:

$$P_n^{(i)} = -\sum_{m=1}^{M} A^{\Sigma}_{mn} T_m^{4(i)}, \quad n = M+1, \ldots, N. \tag{18}$$

The system of equations (17) can be solved using the following rapidly converging iterative method:

$$\vec{T}^{4(j+1)} = \|A\|^{-1}\vec{P}^{(i)} - \|A\|^{-1}\vec{Q}_{\beta}^{*(j)},$$
$$\vec{Q}_{\beta}^{*(j)} = (1-\beta)\vec{Q}_{\beta}^{*(j-1)} + \beta\vec{Q}^{*(j)}, \quad 0 < \beta < 1. \tag{19}$$

The components of the vector $\vec{Q}^{*(j)}$ are determined from the relations:

$$Q_n^{*(j)} = -(\rho C_p)_n V_n \frac{T_n^{(j)} - T_n^i}{\Delta\tau} + \sum_{m=1}^{N} g_{mn} T_m^j + Q_{\text{in},n}^{i+1},$$
$$Q_n^{*(1)} = Q_n^{(i-1/2)}, \quad n = M+1, \ldots, N, \tag{20}$$

and in addition the temperatures of the C zones do not vary during the iteration process, $T_m^j = T_m^i$ for $m \leqslant M$, while the temperatures of the P zones are determined from the relations

$$T_n^{(1)} = T_n^{(i)}, \quad T_n^{(j+1)} = (T_n^{4(j+1)})^{1/4}, \quad m > M. \tag{21}$$

Here the indices j indicate the values of the quantities after the j-th iteration, while the indices i, i + 1/2 correspond to the values of quantities at times τ_i and in the interval $[\tau_i, \tau_{i+1}]$.

If the iteration process (19) converges, then for $j \to \infty$ $T_n^{(j)} \to T_n^{(i+1)}$ for n > M, while (20) goes over into (12). When the matrix $\|A\|$ is independent of temperature and is easily invertible, this iteration process requires $(N - M)^2$ operations at each step.

The convergence of the iterative method (19) with a successful choice of β can be proved rigorously, if the matrix of raidative transfer coefficients $\|A\|$ and the matrix of coefficients of convective transfer of heat between zones $[g_{mn}]$ are positive definite, the first conditions occurs in examining radiative heat transfer in the gray-body approximation with isotropic scattering, while the second occurs if the convective heat transfer in the system occurs due to heat emission and turbulent heat conduction.

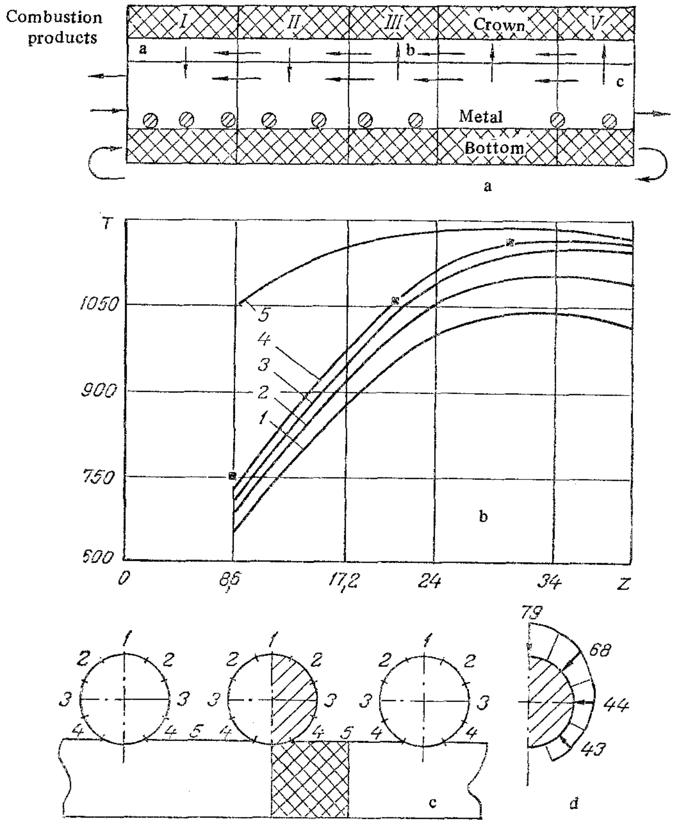

Fig. 1. Simulation of the thermal operation of a ring-shaped furnace: a) geometrical model of the furnace; b) temperature of the metal surface (zone 1) in the process of heating the furnace (curves 1, 2, 3 correspond to 5, 10, and 20 h after the heating begins; 4, 5 are the temperatures of the metal surface and crown of the furnace in the stationary state); the points indicate the experimental values of the temperatures of the metal surface (zone 1); c) separation of the metal surface into zones; d) sketch showing the heat fluxes on the surfaces of the cylindrical parts (kW/m²). T, °C; z, m.

In practice, it turns out that iterations of the form (19) always converge quite rapidly, if the transfer of heat due to radiation predominates in the heat balance of P zones. In this case, it is possible to use values of β in the range 0.5-0.8. As the convective–conductive component increases in the heat balance of P zones, in order for the iteration process to converge, it is necessary to decrease the quantity β, and in this case, the convergence gradually becomes worse and the use of $\beta < 0.1$ is not expedient.

We note that the convergence of the iteration method (19) is nearly independent of the initial approximation, but with an unfortunate choice of the initial values, it can happen that during the iteration $T_n^{4\,(j)} < 0$. For this reason, it is more convenient to replace (21) in practice by the following expression:

$$T_n^{j+1} = \begin{cases} T_n^{(j)}/(2 - T_n^{4(j+1)}/T_n^{4(j)})^{1/4} & \text{for } T_n^{4(j+1)} < T_n^{4(j)}, \\ (T_n^{4(j+1)})^{1/4} & \text{for } T_n^{4(j+1)} \geqslant T_n^{4(j)}, \quad n = M+1, \ldots, N. \end{cases} \tag{22}$$

Relations (22) are empirical and ensure convergence of iterations for any starting approximation; in addition, if $T_n^{j+1} \to T_n^j$, then (22) goes over into (21).

The algorithm proposed was used to construct the dynamic model of heating of cylindrical parts in a ring-shaped furnace. The geometrical model of a flame ring-shaped furnace (Fig. 1a) represents an involute in the form of a parallelepiped, separated in length into five technological regulation zones. Two volume gas zones and seven surface zones were introduced into each technological zone. Zones 1-4 are situated on a metal surface (Fig. 1c), while zones 5, 6, and 7 correspond to the bottom, walls, and crown of the furnace.

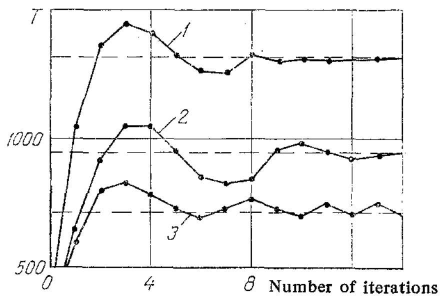

Fig. 2. Convergence of the iterative process when the burners are turned on in a cold furnace: 1, 2, 3) temperature zones a, b, c (Fig. 1a) during the iteration process. T, °K.

In order to determine the selective coefficients of radiative transfer, we used a nine-band model of the radiation spectrum [4]. The crown on the furnace and its walls were modeled by plates consisting of five elements in accordance with the number of technological zones. For each element we constructed a grid consisting of eight nodes, necessary for solving the one-dimensional nonlinear heat-conduction equation. It was assumed that within the technological zone, the parts are heated under identical conditions, so that it is sufficient to determine the temperature field only in a single part and in the adjacent part of the bottom (Fig. 1c). In order to solve the corresponding two-dimensional heat-conduction problem, we constructed a difference grid containing 130 nodes. Thus, the total number of nodes in the difference grid was 740.

The scheme of the gas motion in the furnace, the distribution of the fuel flow over the zones, and the experimental measurements of the temperatures of the metal surface are taken from [5]. On the outer surface of the furnace casing, the temperature was assumed to be fixed and on the metal–bottom boundary, we used boundary conditions of the fourth kind. The temperature of the metal at the inlet to the furnace was assumed to be fixed, while for the bottom, we defined cyclical conditions: the temperature fields at the outlet from the furnace and at the inlet into the furnace are identical.

The stationary thermal regime of the furnace does not depend on the initial conditions. It is convenient to assume that initially the furnace is at a temperature of 0°C and is filled with metal with the same temperature, although this does not correspond to the furnace heating technology. In this case, the time for establishing a stationary state in the furnace is 50 h.

Initially, when the burners are turned on in the cold furnace, the temperature of the gas zones turns out to be much lower than in a real operating furnace. The convergence of the iteration process (19) under these conditions is shown in Fig. 2. It is evident that the lower the temperature of the zone and, therefore, the less important is the heat transfer by radiation in the thermal balance of the zone, the worse the convergence of the iteration process. Under real conditions, 80% of the heat in a given type of furnace is transferred by radiation, while already 2-3 iterations of the form (19) permit solving the system of equations (17) with an error not exceeding 1°K.

The curves showing the heating of the metal, presented in Fig. 1b, correspond to a furnace productivity of G = 54 tons/h with a diameter of the parts equal to 0.17 m and total heat power $b_{\Sigma}Q_n^p$ = 32 MW. It is evident that in the stationary state the computed and experimentally determined temperatures of the metal surface agree well. For this regime of furnace operation, Fig. 1d shows a sketch of the heat flows on the surface of the cylindrical part in the second technological zone.

In conclusion, we note that the calculations carried out demonstrate the quite high efficiency of the algorithm proposed: simulation of heating of parts in a ring-shaped furnace, occurring under real conditions within 1.5 h, requires 10 min of machine time using a ES-1033 computer. This suggests that the expenditures of machine time in using such an algorithm for constructing more detailed models of heating of bodies with complex shape under conditions of radiative–convective heat transfer will be acceptable.

NOTATION

$Q_{rad.n}$ and Q_n, resulting heat flows into zone n due to radiation and due to other types of heat transfer; A_{mn}^{Σ}, selective coefficients of radiative transfer; T_n, temperature of zone n; $\Theta_k(x, y, z, \tau)$, temperature field in the k-th body; $W_{q'\to n}$ and W_n, mass velocities of the gas entering zone n from a neighboring zone q′ and leaving zone n; $\bar{C}_p$, C_p, λ, ρ, average and true heat capacities, thermal conductivity, and density of the medium; Q_n^+, Q_n^-, heat flows gained and lost by the volume zone due to motion of the gas; Q_n^{α}, Q_n^{tr}, heat flows into zone n due to heat transfer and turbulent heat conduction; α_m, $K_{nq'}$, coefficients of heat transfer by convection and turbulent heat conduction; F_m, area of the surface zone m; $F_{nq'}$, area of the boundary of the volume zones n and q'; g_{mn}, coefficients of convective heat transfer; N, total number of zones; M, total number of C zones; $\Delta\tau$, time step; T_n^i, zone temperature at time $\tau_i = \Delta\tau i$; $\Theta_k^{i+1/2}$, temperature field in the interval $[\tau_i, \tau_{i+1}]$; T_n^j, value of the zone temperature after the j-th iterations; $\| A \|$, a $(N - M) \times (N - M)$ matrix; $\vec{T}^i, \vec{P}, \vec{Q}, \vec{Q}^*$, (N − M) dimensional vectors; β, positive constant, $\beta < 1$.

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SOLUTION OF THE STEADY-STATE PROBLEM OF HEAT EXCHANGE AND FLOW OF LUBRICANT IN RADIAL SLIDING BEARINGS WITH SELF-ALIGNING SEGMENTS

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UDC 621.165

The article describes a method based on the use of implicit finite-difference schemes, and it presents the results of the numerical solution of the problem of heat and exchange and flow of lubricant in multisegment radial sliding bearings.

The numerical solution of the problem of liquid flow and heat exchange in radial segmental sliding bearings is based on the well-known assumptions of Reynolds' hydrodynamic theory of lubrication. The physical parameters of oil were taken as constant, and they were determined according to the mean oil temperature in the gap, which was found from the solution of the heat-transfer equation.

The initial system of differential equations describing the intensity of heat transfer of the shaft in radial sliding bearings has the following form in dimensionless values:

$$\frac{\partial}{\partial x}\left(h^3 \frac{\partial p}{\partial x}\right) + \frac{\partial}{\partial z}\left(h^3 \frac{\partial p}{\partial z}\right) = 6\frac{\partial h}{\partial x}, \tag{1}$$

$$u\frac{\partial t}{\partial x} + \left(\frac{v}{h} - u\frac{y}{h}\frac{\partial h}{\partial x}\right)\frac{\partial t}{\partial y} + w\frac{\partial t}{\partial z} = \frac{1}{\mathrm{Pe}h^2}\frac{\partial^2 t}{\partial y^2} + \frac{\mathrm{Ec}}{\mathrm{Re}h^2}\left[\left(\frac{\partial u}{\partial y}\right)^2 + \left(\frac{\partial w}{\partial y}\right)^2\right], \tag{2}$$

$$\mathrm{Nu} = \frac{R/L}{\pi\psi h(1 - t_f)}\int_0^{2\pi}\int_0^{L/R}\left.\frac{\partial t}{\partial y}\right|_{y=0} dxdz. \tag{3}$$

The coordinate Y is reckoned from the surface of the shaft, X from the horizontal axis, and Z from one of the end faces of the bearing.

The boundary conditions for solving the problem had the following form:

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